I_{DDS}/V_P

5.3 TRANSFER CHARACTERISTICS

Derivation

For the BJT transistor the output current I_C and input controlling current I_B were related by beta, which was considered constant for the analysis to be performed. In equation form,

control variable

$$I_C = f(I_B) = \beta I_B \tag{5.2}$$

constant

In Eq. (5.2) a linear relationship exists between I_C and I_B. Double the level of I_B and I_C will increase by a factor of two also.

Unfortunately, this linear relationship does not exist between the output and input quantities of a JFET. The relationship between I_D and V_{GS} is defined by *Shockley's equation:*

control variable

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 \tag{5.3}$$

constants

The squared term of the equation will result in a nonlinear relationship between I_D and V_{GS}, producing a curve that grows exponentially with decreasing magnitudes of V_{GS}.

For the dc analysis to be performed in Chapter 6, a graphical rather than mathematical approach will in general be more direct and easier to apply. The graphical approach, however, will require a plot of Eq. (5.3) to represent the device and a plot of the network equation relating the same variables. The solution is defined by the point of intersection of the two curves. It is important to keep in mind when applying the graphical approach that the device characteristics will be *unaffected* by the network in which the device is employed. The network equation may change along with the intersection between the two curves, but the transfer curve defined by Eq. (5.3) is unaffected. In general, therefore:

The transfer characteristics defined by Shockley's equation are unaffected by the network in which the device is employed.

The transfer curve can be obtained using Shockley's equation or from the output characteristics of Fig. 5.10. In Fig. 5.15 two graphs are provided, with the vertical

William Bradford Shockley (1910–1989), co-inventor of the first transistor and formulator of the "field-effect" theory employed in the development of the transistor and FET. (Courtesy of AT&T Archives.)

Born: London, England
PhD Harvard, 1936
Head, Transistor Physics Department–Bell Laboratories
President, Shockley Transistor Corp.
Poniatoff Professor of Engineering Science at Stanford University
Nobel Prize in physics in 1956 with Drs. Brattain and Bardeen

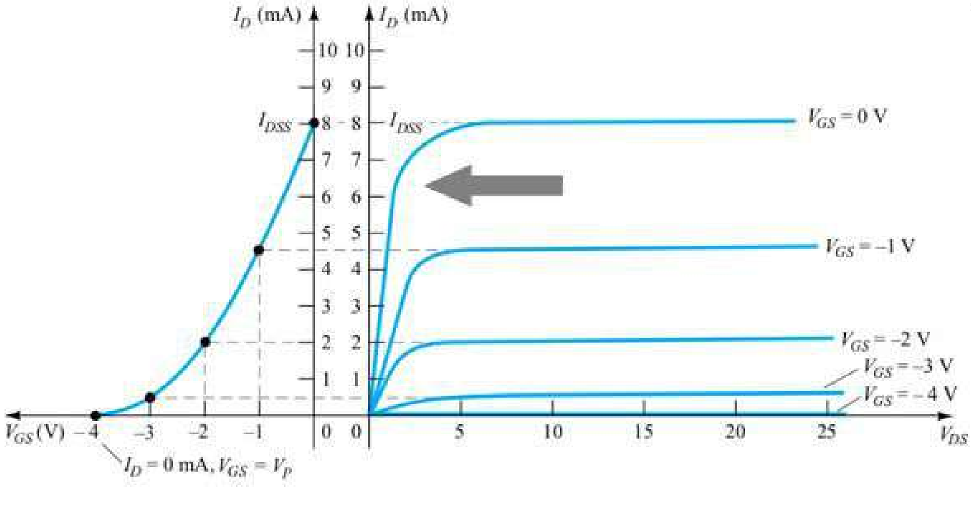

Figure 5.15 Obtaining the transfer curve from the drain characteristics.

I_{DDS}/V_P

scaling in milliamperes for each graph. One is a plot of I_D versus V_{DS}, while the other is I_D versus V_{GS}. Using the drain characteristics on the right of the "y" axis, a horizontal line can be drawn from the saturation region of the curve denoted $V_{GS} = 0$ V to the I_D axis. The resulting current level for both graphs is I_{DSS}. The point of intersection on the I_D versus V_{GS} curve will be as shown since the vertical axis is defined as $V_{GS} = 0$ V.

In review:

When $V_{GS} = 0$ V, $I_D = I_{DSS}$.

When $V_{GS} = V_P = -4$ V, the drain current is zero milliamperes, defining another point on the transfer curve. That is:

When $V_{GS} = V_P$, $I_D = 0$ mA.

Before continuing, it is important to realize that the drain characteristics relate one output (or drain) quantity to another output (or drain) quantity—both axes are defined by variables in the same region of the device characteristics. The transfer characteristics are a plot of an output (or drain) current versus an input-controlling quantity. There is therefore a direct "transfer" from input to output variables when employing the curve to the left of Fig. 5.15. If the relationship were linear, the plot of I_D versus V_{GS} would result in a straight line between I_{DSS} and V_P. However, a parabolic curve will result because the vertical spacing between steps of V_{GS} on the drain characteristics of Fig. 5.15 decreases noticeably as V_{GS} becomes more and more negative. Compare the spacing between $V_{GS} = 0$ V and $V_{GS} = -1$ V to that between $V_{GS} = -3$ V and pinch-off. The change in V_{GS} is the same, but the resulting change in I_D is quite different.

If a horizontal line is drawn from the $V_{GS} = -1$ V curve to the I_D axis and then extended to the other axis, another point on the transfer curve can be located. Note that $V_{GS} = -1$ V on the bottom axis of the transfer curve with $I_D = 4.5$ mA. Note in the definition of I_D at $V_{GS} = 0$ V and -1 V that the saturation levels of I_D are employed and the ohmic region ignored. Continuing with $V_{GS} = -2$ V and -3 V, the transfer curve can be completed. It is the transfer curve of I_D versus V_{GS} that will receive extended use in the analysis of Chapter 6 and not the drain characteristics of Fig. 5.15. The next few paragraphs will introduce a quick, efficient method of plotting I_D versus V_{GS} given only the levels of I_{DSS} and V_P and Shockley's equation.

Applying Shockley's Equation

The transfer curve of Fig. 5.15 can also be obtained directly from Shockley's equation (5.3) given simply the values of I_{DSS} and V_P. The levels of I_{DSS} and V_P define the limits of the curve on both axes and leave only the necessity of finding a few intermediate plot points. The validity of Eq. (5.3) as a source of the transfer curve of Fig. 5.15 is best demonstrated by examining a few specific levels of one variable and finding the resulting level of the other as follows:

Substituting $V_{GS} = 0$ V gives

$$\text{Eq. (5.3):}\quad I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$= I_{DSS}\left(1 - \frac{0}{V_P}\right)^2 = I_{DSS}(1 - 0)^2$$

and

$$I_D = I_{DSS}\big|_{V_{GS} = 0\text{ V}} \tag{5.4}$$

Substituting $V_{GS} = V_P$ yields

$$I_D = I_{DSS}\left(1 - \frac{V_P}{V_P}\right)^2$$
$$= I_{DSS}(1 - 1)^2 = I_{DSS}(0)$$

$$I_D = 0 \text{ A}\big|_{V_{GS} = V_P} \tag{5.5}$$

For the drain characteristics of Fig. 5.15, if we substitute $V_{GS} = -1$ V,

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$
$$= 8 \text{ mA}\left(1 - \frac{-1 \text{ V}}{-4 \text{ V}}\right)^2 = 8 \text{ mA}\left(1 - \frac{1}{4}\right)^2 = 8 \text{ mA}(0.75)^2$$
$$= 8 \text{ mA}(0.5625)$$
$$= \mathbf{4.5 \text{ mA}}$$

as shown in Fig. 5.15. Note the care taken with the negative signs for V_{GS} and V_P in the calculations above. The loss of one sign would result in a totally erroneous result.

It should be obvious from the above that given I_{DSS} and V_P (as is normally provided on specification sheets) the level of I_D can be found for any level of V_{GS}. Conversely, by using basic algebra we can obtain [from Eq. (5.3)] an equation for the resulting level of V_{GS} for a given level of I_D. The derivation is quite straight forward and will result in

$$V_{GS} = V_P\left(1 - \sqrt{\frac{I_D}{I_{DSS}}}\right) \tag{5.6}$$

Let us test Eq. (5.6) by finding the level of V_{GS} that will result in a drain current of 4.5 mA for the device with the characteristics of Fig. 5.15.

$$V_{GS} = -4 \text{ V}\left(1 - \sqrt{\frac{4.5 \text{ mA}}{8 \text{ mA}}}\right)$$
$$= -4 \text{ V}(1 - \sqrt{0.5625}) = -4 \text{ V}(1 - 0.75)$$
$$= -4 \text{ V}(0.25)$$
$$= \mathbf{-1 \text{ V}}$$

as substituted in the above calculation and verified by Fig. 5.15.

Shorthand Method

Since the transfer curve must be plotted so frequently, it would be quite advantageous to have a shorthand method for plotting the curve in the quickest, most efficient manner while maintaining an acceptable degree of accuracy. The format of Eq. (5.3) is such that specific levels of V_{GS} will result in levels of I_D that can be memorized to provide the plot points needed to sketch the transfer curve. If we specify V_{GS} to be one-half the pinch-off value V_P, the resulting level of I_D will be the following, as determined by Shockley's equation:

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$
$$= I_{DSS}\left(\frac{1 - V_P/2}{V_P}\right)^2 = I_{DSS}\left(1 - \frac{1}{2}\right)^2 = I_{DSS}(0.5)^2$$
$$= I_{DSS}(0.25)$$

I_{DDS}/V_P

and $$I_D = \frac{I_{DSS}}{4}\bigg|_{V_{GS} = V_P/2} \tag{5.7}$$

Now it is important to realize that Eq. (5.7) is not for a particular level of V_P. It is a general equation for any level of V_P as long as $V_{GS} = V_P/2$. The result specifies that the drain current will always be one-fourth of the saturation level I_{DSS} as long as the gate-to-source voltage is one-half the pinch-off value. Note the level of I_D for $V_{GS} = V_P/2 = -4\text{ V}/2 = -2\text{ V}$ in Fig. 5.15.

If we choose $I_D = I_{DSS}/2$ and substitute into Eq. (5.6), we find that

$$V_{GS} = V_P\left(1 - \sqrt{\frac{I_D}{I_{DSS}}}\right)$$

$$= V_P\left(1 - \sqrt{\frac{I_{DSS}/2}{I_{DSS}}}\right) = V_P(1 - \sqrt{0.5}) = V_P(0.293)$$

and $$V_{GS} \cong 0.3V_P\big|_{I_D = I_{DSS}/2} \tag{5.8}$$

Additional points can be determined, but the transfer curve can be sketched to a satisfactory level of accuracy simply using the four plot points defined above and reviewed in Table 5.1. In fact, in the analysis of Chapter 6, a maximum of four plot points are used to sketch the transfer curves. On most occasions using just the plot point defined by $V_{GS} = V_P/2$ and the axis intersections at I_{DSS} and V_P will provide a curve accurate enough for most calculations.

TABLE 5.1 V_{GS} versus I_D Using Shockley's Equation

V_{GS}	I_D
0	I_{DSS}
0.3 V_P	$I_{DSS}/2$
0.5 V_P	$I_{DSS}/4$
V_P	0 mA

EXAMPLE 5.1 Sketch the transfer curve defined by $I_{DSS} = 12$ mA and $V_P = -6$ V.

Solution

Two plot points are defined by

$$I_{DSS} = \mathbf{12\ mA} \quad \text{and} \quad V_{GS} = \mathbf{0\ V}$$

and $$I_D = \mathbf{0\ mA} \quad \text{and} \quad V_{GS} = \boldsymbol{V_P}$$

At $V_{GS} = V_P/2 = -6\text{ V}/2 = \mathbf{-3\ V}$ the drain current will be determined by $I_D = I_{DSS}/4 = 12\text{ mA}/4 = \mathbf{3\ mA}$. At $I_D = I_{DSS}/2 = 12\text{ mA}/2 = \mathbf{6\ mA}$ the gate-to-source voltage is determined by $V_{GS} \cong 0.3V_P = 0.3(-6\text{ V}) = \mathbf{-1.8\ V}$. All four plot points are well defined on Fig. 5.16 with the complete transfer curve.